Strategic Data Acquisition and Price Competition

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Abstract

Leveraging the power of modern data analytics and the increasing access to consumer data, businesses can now infer consumer preferences, enabling them to personalize advertising and implement differential pricing strategies. However, the consequences of determining which consumer information to acquire become unclear when firms engage in competition. To explore the strategic implications of data acquisition choices on market competition, I present a two-stage duopoly model. In the first stage, firms decide which consumer characteristics they aim to learn, and in the second stage, both firms engage in costly advertising with the gathered information. In contrast to the monopoly benchmark, where the monopolistic firm never acquires partial information, I demonstrate that under competition, equilibria exist where both firms strategically acquire distinct consumer characteristics. My findings reveal a non-monotonic effect of higher information costs on firms' profits, wherein profits increase when information is inexpensive but decrease when the expense becomes relatively high. Moreover, as the cost of information acquisition rises, the consumer surplus generally experiences a decline.

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1 Introduction

The advent of internet platforms, exemplified by industry giants such as Google, Amazon, and Facebook, has ushered in an era marked by an unparalleled accumulation of personal data. The availability of these data, valuable for informed business decision-making, is transforming the landscape. Importantly, while current technology is already advanced, the full utilization of these data and algorithms may not be available now but is likely to emerge in the near future. Once these capabilities become accessible, firms that were previously confined to disseminating messages to diverse consumer groups will have the potential to acquire data on relatively homogenous groups of individuals through intermediary information brokers[1].

In this landscape, cutting-edge machine learning algorithms, coupled with access to consumer data, empower businesses to acquire detailed individual information on tastes. This newfound capability enables businesses to provide customized price offers tailored to each customer's specific tastes. However, it's crucial to note that the acquisition of such valuable information could come at a non-negligible cost. Moreover, the individual data acquisition choices made by one firm may have far-reaching effects on the strategies of others, introducing non-trivial impacts on the competition.

In the absence of competition and the associated cost of information acquisition, the welfare implications of consumer data are unambiguous; information that allows the monopoly for price discrimination weakly raises total profit. This is simply a consequence of Blackwell (1951), Blackwell (1953) that information is always valuable for a single receiver. In this context, Bergemann, Brooks and Morris (2015) study the third-price discrimination problem under different information structures. All welfare pairs can be realized through some information structures. By varying the information structures exogenously, they also show that the more information available to the monopoly firm, the finer market segmentation it leads to, hence a higher producer surplus.

This paper intends to consider a setting where both firms endogenously acquire relevant information structures before market competition. The choice of information structures entails collecting the consumer data and learning the content behind the data. In this setting, it is unclear whether more information is still better for firms.

Motivated by these concerns, this paper develops a two-stage duopoly model that involves information acquisition and advertising with price competition. In contrast to the majority of theoretical duopoly models categorizing individual consumers based on reservation prices, this model classifies them according to observable characteristics. The valuation of the product correlates with these observable characteristics within the same set, and this distribution is known to both firms. Specifically, individual valuation is a binary value (0 or 1) determined

[1] The strategic incentive of selling information to firms by the data brokers is abstracted away.

by its characteristics and the 'values' associated with these characteristics, representing an interested or uninterested buyer.

The product valuation remains uncertain for firms without additional information. Consequently, in the first stage, firms are permitted to acquire more information, albeit at a cost, provided the improved information leads to higher profits in later competition. Information acquisition involves understanding the 'values' of the observed characteristics. If a firm knows the 'values' of all characteristics, it perfectly learns the consumer's valuation. Therefore, I model information choice as a subset of all observable characteristics. To make the model more tractable, I restrict there are only two relevant consumer characteristics and I assume the information choices are public to both firms after they made the decisions.

In the second stage, while each firm possesses knowledge of potential consumers' characteristics, the process of reaching them with personalized price advertisements incurs costs. This cost encompasses payments to data brokers for delivering the name and contact information of a consumer with the relevant characteristics. Additionally, it includes the costs associated with preparing and delivering individualized offers. These offers could take the form of text messages, emails, and personalized ads on websites or apps, as opposed to mass marketing. In line with Butters (1978) and Stahl (1989), a consumer remains unaware that a product is available unless she receives an advertised offer from the selling firm. When presented with multiple offers, she then selects the one that maximizes her consumer surplus.

The setting of competition via costly advertising is similar to Anderson, Baik and Larson (2015), but differs in two aspects. The information acquisition component is embedded in advertising, Anderson et al. (2015), as they assume by paying advertising costs, firms perfectly know consumers' valuations. The second difference is they allow different (deterministic) product valuations, while I assume both products are homogenous and the valuation is a priori uncertain to both firms.

Under this framework, the "value" of each characteristic can be simply viewed as the coefficient attached to that characteristic. Then the question of what information to acquire could be viewed as which coefficient to learn. In the big data era, data analysts might recast the above setting as a typical classification problem with variable selection. Such algorithms aim to pick the variables that help to best predict the consumer's valuation. The result of this paper is quite different from the traditional statistical exercise of variable selection in the sense that the data acquisition decision in my model speaks to the strategic competition between the firms.

To illustrate this strategic incentive, suppose, in the extreme case, that firm A knows that firm B decides to learn the consumer's valuation perfectly by acquiring all characteristics. Since all information choices are publicly observable, whatever information choice firm A chooses, firm B knows exactly firm A's realized belief, which is equivalent to knowing firm

A's lowest possible price to advertise. Anticipating the consequence, the best response for a firm A is thus no learning, which cannot be a result of variable selection.

I first analyze the benchmark case of monopoly. In this situation, without any competition, the monopoly firm can always charge a high price in the second stage. When the information cost is positive but not too large, the monopoly prefers full learning whenever the advertising cost is intermediate, and it prefers no learning in the other region of the advertising cost. This result relies on the assumption that the marginal cost of an additional characteristic dimension is constant. The economic intuition behind is not complex. The first key observation is that when the monopoly advertises, the advertising price is 1, as without competition, giving consumers more surplus is not as profitable. Secondly, note that the cost of advertising reflects the extensive margin of the cost of information, i.e. the cost of sending ads to uninterested buyers. When the advertising cost is low, the punishment is not severe and it is better for the monopoly firm not to learn and advertise at a price of 1. When the advertising cost rises, the value of information increases, and the firm now prefers full learning. When advertising becomes higher than the value of information given by full learning, the monopoly firm is better off with no learning and no advertising. In the monopoly benchmark, partial learning, i.e., learning some of the characteristics, is never optimal. For each information choice, the profit is weakly decreasing. As a result, the optimal value is weakly decreasing in advertising cost.

Under the duopoly setting, the second-stage price competition is more complex. To better describe the equilibria, I characterize all equilibria in terms of the firm's belief about consumer valuations. Essentially, each pair of information choices generates a joint distribution of firms' beliefs about consumers' valuation for a given consumer group. I will refer to this joint distribution as an information structure. Note that when one firm's information choice is a subset of the other firm, its realized belief is available to its rival.

Given a potential consumer with specific observable characteristics, firms face simultaneous decisions on whether to advertise to her and, if so, what individualized price to offer. In an information structure where each firm has some private information, i.e., the information choices are non-empty and non-overlapped, the competition resembles a common-value auction with an entry cost. In this case, when the entry cost is high, only confident firms are likely to advertise, mitigating concerns about the Winner's curse. Conversely, with a low entry cost, a firm with lower confidence may choose to advertise. To avoid potential losses attributed to the Winner's curse, this firm opts for a high price, increasing the likelihood of securing a sale only when its rival is confident. Essentially, the pricing strategy in this situation is non-monotone in the individual firm's belief. Due to the nature of the cutoff strategy, each firm earns a strictly positive profit in expectation.

In other information structures where one firm has more information than its opponent, equilibria involve price dispersion and the decision to advertise. Similar to Anderson et al. (2015), in equilibria, the firm that has the advantage earns a strictly positive profit, and the other firm earns zero profit.[2] Notably, whenever both firms' information choices are identical, they earn zero profit, resembling the standard Bertrand competition with two identical firms.

To compare with the benchmark monopoly case, fix a positive information cost that is not too high. First note that there are multiple equilibria. One interesting feature of some of the equilibrium payoffs is that sometimes advertising costs relax competition. Roughly speaking this situation occurs because, in some of the market segments, the equilibrium strategy implies firm A is well-informed of consumers' valuation, while the other firm is completely ignorant, and thus relies on only prior information. With loss of generality, suppose the buyer is interested in the product. Because the advertising cost is low, the ignorant firm would enter with some probability. Whenever the ignorant firm advertises, its pricing strategy is a mixed strategy. Due to uncertainty, the ignorant firm's price must be weakly higher than the advertising cost. Earning a zero profit implies that the ignorant firm's lowest price barely covers the advertising cost. It turns out this lowest price is linear in the advertising cost. In equilibrium well-informed advertises with certainty. Pricing strategy of the informed firm is also a mixed strategy, and the lower support matches. Because posting at this price wins the sale with probability 1, this price equals the revenue. Therefore, as the advertising cost rises, the linearity of the price implies that price competition is softened.

To see the effect of information costs on the equilibrium, I find that for the equilibrium where each firm is mixing between full learning and distinct learning, profits increase in information costs. As information costs rise, firms put more weight on distinct learning, as the marginal disutility of full learning decreases faster than it of distinct learning. The consumers, however, suffer, as they are better off when firms choose the same information choice, which induces perfect price competition. As, information cost increases, profit decreases because the value of information cannot justify its cost, and therefore no firm learns. This induces a zero profit, and the consumer, at this point, benefits from the perfect price competition and enjoys a lower surplus than when information is cheap.

Related Literature

This paper is related to the classical literature on informative advertising. Seminal works, including Butters (1978) and Stahl (1989), have traditionally defined informative advertising as situations where consumers learn about both products and prices through advertisements; in contrast, in this paper, there is an additional information acquisition component

[2] n Anderson et al. (2015), a firm with an advantage is characterized by a higher product valuation than its rival. In my model, this advantage specifically refers to an information advantage, meaning its rival's information choice is a strict subset of its own.

before advertising. Segmentating based on observable characteristics, different information acquisition strategies enable advertising different prices in different segments. Galeotti and Moraga-González (2008) studies advertising in a duopoly with homogenous products. In their model, consumers are segmented by characteristics, while characteristics are not correlated with product valuation, and thus, information acquisition plays no role. Chen and Iyer (2002) study personalized pricing when firms first need to invest for addressability; since they consider spatial competition, location is correlated with the consumer's preference. In contrast, this paper considers the competition for homogenous products.

The concept of personalized pricing has gained renewed significance with advancements in information technology. Competitive personalized pricing is explored in recent works such as Baik and Larson (2023) and **?**, both utilizing a general discrete-choice framework. In a different vein, Ali, Lewis and Vasserman (2022) investigates personalized pricing under privacy concerns, where firms don't directly acquire information but rather rely on information voluntarily disclosed by consumers through their privacy choices.

There is also growing research on the strategic use of data under competition. In Iyer and Ke (2023), algorithmic targeting is investigated within a framework where the bias-variance trade-off plays a crucial role in determining the targeted consumer segment. Similarly, Feng, Gradwohl, Hartline, Johnsen and Nekipelov (2022) studies the strategic choice of algorithms under a competitive environment.

Outline

The rest of this paper is organized as follows: Section 2 introduces the two-stage model. Section 3 states the benchmark result under monopoly. Section 4 characterizes the equilibrium of the second-stage competition under different information structures. Section 5 discusses the equilibrium outcome of the strategic information choices in the first stage. Section 6 concludes.

2 Model

Two sellers sell a homogenous product to a unit measure of buyers. Each buyer has two observable attributes $(x_1, x_2) \in X = \{(1,1),(1,0),(0,1)\}$, where each attribute takes binary values. For simplicity, I ignore the market segment $(x_1, x_2) = (0,0)$ and assume that each of the rest market segments has a size of $1/3$. An individual buyer's valuation of the product is determined by a real-valued score θ, which only depends on the buyer's characteristics. Specifically, each buyer has a valuation $v = 1$ if $\theta > 0$ and $v = 0$ otherwise. Without loss of generality, v is normalized to be 1. In our setup, the score is linear in characteristics, i.e., $\theta = \beta_1 x_1 + \beta_2 x_2$. The score θ is not directly observable by both sellers a priori. Both sellers have a common prior belief of β_1, β_2 such that each β_i follows a standard normal distribution, and they are independent of each other.

In the first stage of the game, each firm can privately learn a subset J of $\{\beta_1, \beta_2\}$ with a cost of t per attribute. At the end of the first stage, each $\beta \in J$ is revealed to the firm. This modeling choice reflects the idea that in the era of big data, purchasing data is tantamount to learning the unknown state, and the learning cost can be viewed as data acquisition expenses.

Following the learning decision, I assume that both firms observe their opponents' learning choices. Subsequently, both firms compete via advertising. Since consumer characteristics are observable, firms can employ different advertising strategies for different market segments. Competition via advertising takes the following form. Each firm could advertise its price p, incurring a cost of $c < 1$, or choose not to advertise. Buyer cannot purchase if she receives no advertisement. If a buyer receives only one ad, a purchase is made if the surplus generated is above zero, which is the value of the outside option. If a buyer receives only two ads, the ad with a lower price is accepted if it generates a positive surplus.

The timing of the game is as follows:

1. **Information Acquisition Decision**: Each firm makes its information acquisition choice. Opting to learn leads to the realization of the relevant states. After learning occurs, the realized $\beta's$ are revealed to firms.
2. **Engaging in Price Competition**: Both firms observe the information acquisition decision made by their rival. Informed by their private learning results and the competitor's information collection choice, each firm simultaneously decides whether to advertise with a cost of c and what price to post if advertising for each market segment.
3. **Consumer's Purchasing Decision**: If a buyer receives any ads, she can choose the firm with the lowest price (randomly if prices are equal) or take the outside option.

2.1 Interim expected values and strategy

To simplify the analysis, in our setup, the interim expected value of the product is a sufficient statistic. Since the valuation is binary, the interim expected value is the belief about the product valuation being 1. Therefore, without loss of generality, define the $T_i = [0, 1]$ as the signal space of the beliefs for firm i. Any information structure induces a distribution of beliefs $q_i : J_i \times X \to \triangle(T_i)$ for market segment x. Note that the support could be either continuous or discrete. For example, if firm i does not learn any characteristics, the belief is just 1/2 with probability 1. If firm i learns both characteristics, then q_i is supported on 0 and 1 with equal probability. For different market segments, the same information structure can induce different beliefs. For instance, fix the information structure of just learning β_1. In the market segment $x = (1, 1)$, the support is $(0, 1)$, as β_1 could be any real number.

However, for the market segment $x = (1, 0)$, valuation is 1 if and only $\beta_1 > 0$. Hence, the belief distribution is the same as in the full learning case.

Since firms are allowed to adopt different advertising strategies for different market segments, based on the signal realization. Formally, I define the advertising strategy for firm i by $\gamma_i : T_i \times X \to \triangle(\{0, 1\})$, and the pricing strategy: $p_i : T_i \times X \to \triangle([0, 1])$. If γ_i and p_i are pure strategies, I will abuse notation slightly by writing $\gamma_i(t_i, x)$ and $p_i(t_i, x)$. Let $\sigma_i(t_i, x) = (\gamma_i(t_i, x), p_i(t_i, x))$. Then, the profit in the second stage for segment x is

$$\pi_i(J_i, J_{-i}, \sigma_i, \sigma_{-i}, x) = \gamma_i \left[\mathbb{P}(v = 1|x)\mathbb{P}(p_i \leq p_{-i}) - c\right],$$

where $\mathbb{P}(v = 1)$ is determined by the joint distribution of (q_i, q_{-i}). In the second stage, the relevant solution concept is Bayes Nash equilibrium. The strategy profile (σ_A, σ_B) is a Bayes Nash equilibrium if and only of $\pi_i(J_i, J_{-i}, \sigma_i, \sigma_{-i}, x) \geq \pi_i(J_i, J_{-i}, \sigma_i', \sigma_{-i}, x)$. The total payoff from the second stage is the average of the profits across each segment. The equilibrium concept in the first stage is the standard Nash equilibrium.

3 Monopoly Benchmark

I solve the game by backward induction. Instead of using the realized β's, it is without loss to consider the belief that is induced by β's. Let $q_J|x$ be the belief of the monopoly with information choice J in market segment x. Note that for the same information choice, the belief of the buyer's valuation varies with the market segment, i.e., knowing $beta_1$ completely pins down the valuation of buyer with characteristics (1,0). In the second stage of the game, the monopoly posts a price of 1 whenever the posterior belief exceeds c and chooses not to advertise otherwise. The profit for the aggregate market is

$$\Pi(J) = \left[\sum_{x \in X} \frac{1}{3}\, \mathbb{E}_{q_J|x}[\max(q_J - c, 0)|x]\right] - t|J|,$$

where the expectation is taken over the distribution of beliefs given J. Our first result shows that the monopoly firm never chooses to learn one characteristic.

Proposition 1. *For $0 \leq t \leq 1/8$, the monopoly firm chooses to learn both characteristics if $c \in [4t, 1 - 4t]$ and chooses not to learn any characteristics otherwise. For $t \geq 1/8$, it is always optimal for the monopoly firm not to learn.*

Proof of this result, and all others omitted from the text, may be found in Appendix. The outline of the proof goes as follows. Let $VOI(J_1, J_2)$ denote the value of information between information choice J_1 and J_2 where J_1 is more informative than J_2. Under the linear cost structure, the $VOI(\{\beta_1, \beta_2\}, \{\beta_1\}) \geq VOI(\{\beta_1\}, \emptyset)$. Let $t > 0$ be the information cost of one characteristic. Since the information cost is the same for the above VOIs, the above inequality implies that whenever $\{\beta_1\}$ dominates $\{\beta_1, \beta_2\}$, $\{\beta_1\}$ is dominated by $\emptyset$, and whenever $\{\beta_1\}$ dominates $\emptyset$, $\{\beta_1\}$ is dominated by the $\{\beta_1, \beta_2\}$. Hence, by symmetry, learning only one characteristic is never optimal. It follows that there exists a $c_1 < c_2$ such that β_1 is optimal whenever $c \in (0, c_1) \cup (1 - c_1)$ and $\{\beta_1, \beta_2\}$ is optimal whenever $c \in (c_2, 1 - c_2$. Notice that

$$VOI(\{\beta_1, \beta_2\}, \emptyset) = VOI(\{\beta_1, \beta_2\}, \{\beta_1\}) + VOI(\{\beta_1\}, \emptyset) \leq 2VOI(\{\beta_1, \beta_2\}, \{\beta_1\})$$

Since information cost is linear, it implies there exists a $c_3 \in (c_1, c_2)$ such that β_1 is optimal whenever $c \in (0, c_3) \cup (1 - c_3)$ and $\{\beta_1, \beta_2\}$ is optimal whenever $c \in (c_3, 1 - c_3)$.

The linear cost and value of information among the information choices rule out the case where learning a single characteristic is sometimes optimal. For low information cost t, Proposition 1 says learning both features only happens when advertising cost is intermediate. Intuitively, advertising cost reflects the cost of making mistakes, i.e., sending an ad to a consumer not interested in the product. Consider the extreme case where $c = 0$. In this situation, the value of information complete information is zero because there is no punishment for making mistakes. For a low level of c, because punishment is not severe, the value of information cannot be covered by the information cost $2t$. As punishment becomes more severe, the value of information exceeds the information cost, and the monopoly would rather learn both characteristics. Note that the expected payoff of not learning is $1/2$. When $c > 1/2$, if the monopoly chooses not to learn, it stops advertising. Therefore, the value of information only contains the surplus from the correct decision to advertise. This surplus shrinks whenever $c > 1/2$. Since the information cost is sunk, when c is too high, the surplus from making perfect decisions cannot cover the information cost. It is then optimal to choose not to learn at stage 1.

4 Stage 2: Advertising and Price Competitoin

In this section, I will discuss each firm's advertising strategy, given the acquired information in the first stage. Different data acquisition decisions in the first stage lead to different information structures in the second stage of the game. Roughly speaking, there are three cases: (a) each firm has non-overlapped information,i.e. $J_A = \{\beta_1$ and $J_B = \{\beta_2\}$ or vise versa, (b) both firms have the same information, i.e. $J_A = J_B$, and (c) one firm's information is contained in its rival's acquired information, i.e. $J_A \subseteq J_B$. Without loss of generality, I will first characterize the market outcome in each case within the market segment $x = (1, 1)$, as analysis for other market segments would fall into one of the cases.[3]. The complete characterization of all the market segments will be provided subsequently. To ease the notational burden, I will assume $x = (1,1)$ and drop the notation for x throughout Section 4.1 to Section 4.3.

4.1 Both firms have non-overlapped information

I first introduce a handy lemma that describes the joint distribution of beliefs under the distinct learning information structure.

Lemma 1. *Suppose firm A learns β_1 and firm B learns β_2. Let q_A and q_B be the respective beliefs in the market segment $x = (1,1)$. Then q_A and q_B are i.i.d. uniform $[0,1]$ distributions. Moreover, consumers in the market segment have a valuation of 1 if and only if $q_A + q_B \geq 1$.*

Proof. The belief of firm A is :

$$q_A(\{\beta_1\}) = \mathbb{P}(\beta_1 + \beta_2 \geq 0|\beta_1) = \mathbb{P}(\beta_2 \geq -\beta_1) = 1 - \Phi(-\beta_1),$$

where Φ denotes the CDF of the standard normal distribution. The distribution of q_A can be therefore calculated by

$$\begin{aligned}\mathbb{P} = (q_A \leq x) &= \mathbb{P}(1 - \Phi(-\beta_1) \leq x) \\ &= \mathbb{P}(\Phi(\beta_1) \leq x) \\ &= \mathbb{P}(\beta_1 \leq \Phi^{-1}(x) \\ &= x,\end{aligned}$$

It follows that q_A is a uniform distribution on [0,1]. Because β_1 and β_2 are i.i.d, q_B also follows a uniform [0, 1] distribution, and it is independent of q_A. For the last claim, note that

$$q_A(\{\beta_1\}) + q_B(\{\beta_2\}) \geq 1 \iff G(\beta_1) + (1 - G(-\beta_2)) \geq 1 \iff \beta_1 + \beta_2 \geq 0.$$

∎

In this situation, the competition is similar to the setting of common value auctions with entry. I will consider a symmetric Bayes Nash equilibrium. By the Lemma 1, it is without loss to consider the support of $\mathcal{T}_i = (0,1)$.[4] Suppose each firm would enter the market if

[3] For example, if firm A learned β_1 and firm B learned β_2, then in market segment $x = (1,0)$, firm A would have complete information and firm B had no information. The analysis for this situation would be similar to the case where firm A learns both β's while firm B does not learn in the market segment $x = (1,1)$

[4] Since by Lemma 1 each firm's beliefs are independent, it is sufficient to consider the first-order belief.

their belief is higher than some threshold $\tilde{q} > 0$. This threshold depends on the advertising cost c, as no firm should enter if $c = 1$ and both firms should enter if $c = 0$. Since the entire information structure is known to both firms, each must consider the information winning conveys.

Proposition 2. *Define the function*

$$\psi(x) = \begin{cases} x \exp(-1), & \text{if } x \leq 1/2, \\ x \exp\left(1 - \frac{1}{x}\right)), & \text{if } x > 1/2. \end{cases}$$

ψ is a one-to-one and onto function from $[0,1]$ to $[0,1]$. Define $\tilde{q} = \psi^{-1}(c)$.

(i) If $\tilde{q} \geq 1/2$, then there exists a unique symmetric equilibrium that is strictly increasing when the belief is above the threshold

$$p_i(q) = \begin{cases} \exp\left(-\dfrac{1-q}{\tilde{q}}\right), & \text{if } q \geq \tilde{q}, \\ 0, & \text{otherwise.} \end{cases}$$

$$\gamma_i(q) = \begin{cases} 1, & \text{if } q \geq \tilde{q}, \\ 0, & \text{otherwise.} \end{cases}$$

(ii) If $\tilde{q} < 1/2$, then there exists an equilibrium, which is symmetric around 1/2 in $(\tilde{q}, 1-\tilde{q})$ and equal to the above pricing strategy in $[1-\tilde{q}, 1]$:

$$p_i(q) = \begin{cases} \dfrac{B}{q}, & \text{if } \tilde{q} \leq q \leq \frac{1}{2}, \\ \dfrac{B}{1-q}, & \text{if } \frac{1}{2} \leq q \leq 1-\tilde{q}, \\ \exp\left(-\dfrac{1-q}{\tilde{q}}\right), & \text{if } q \geq 1-\tilde{q} \\ 0, & \text{otherwise.} \end{cases}$$

$$\gamma_i(q) = \begin{cases} 1, & \text{if } q \geq \tilde{q}, \\ 0, & \text{otherwise.} \end{cases}$$

where

$$B = \frac{\tilde{q}}{e}$$

(iii) The equilibrium profits for both firms are stricyly positive.

The first part of the result is that if the advertisement threshold is above 1/2, a unique symmetric, strictly increasing equilibrium exists. Because low-priced ad wins the competition and the threshold strategy, this equilibrium features both the Winner's blessing and the Winner's curse. When the rival firm advertises, winning the competition implies the buyer's valuation is 1, which suggests Winner's curse. When the rival firm does not advertise, the opponent firm's belief may be low, and thus winning incurs a loss.

The behavior differs with the threshold below and above 1/2 for several reasons. First, note that from Lemma 1, firms prefer not to advertise if the sum of the beliefs is less than 1. When the threshold is above 1/2, the Winner's curse is not so severe that there is still a positive probability the buyer's valuation is 1. Suppose the threshold is below 1/2, and the firm's belief is slightly greater than the threshold. If the firm wins when its rival does not advertise, winning induces a loss, so the Winner's curse is more severe. To offset the effect of the Winner's curse, in equilibrium, firms would raise the price to ensure that when the firm wins with its rival also advertising, its rival has a sufficiently high belief. Therefore, the equilibrium strategy cannot be strictly increasing when the firm chooses to advertise.

Since in the equilibrium there is a strictly positive probability where buyer is interested in the product and the firm B does not advertise, the equilibrium prfoit must be strictly positive when the firm A's belief is above the threrhold. By symmetry, firm must earn a positive profit in equilibrium.

4.2 Two firms have the same information

In this case, both firms have the same belief distribution $q_A = q_B = q$, and this is common knowledge. It could happen when both firms learn two attributes (q is supported on either 0 or 1), both firms learn a common attribute q is supported on (0,1), or both firms do not learn ($q = 1/2$ with probability 1). Due to the advertising cost and the undercutting incentives in the standard Bertrand competition, no pure strategy exists. As in the standard Bertrand competition game with the common information structure, the profits to both firms are zero.

Proposition 3. *Suppose $J_A = J_B$. Let q be the realized belief. In equilibrium, if $q < c$, neither firm would advertise. For $q \geq c$, both firms advertise with probability $1 - c/q$, and both firms send price offers according to the following distribution*

$$H(p;q) = \begin{cases} 0, & \text{if } p < c/q, \\ \dfrac{1 - \dfrac{c}{pq}}{1 - \dfrac{c}{q}}, & \text{if } c/q \leq p < 1 \\ 1, & \text{if } p \geq 1. \end{cases}$$

The expected profit for each firm is zero.

The mixed strategy of pricing is an atomless distribution. As is typical with mixed pricing strategy equilibrium in competition, both firms' price distributions are in the truncated Pareto family with shape parameter 1. Many well-known papers derive Pareto distributions from their mixed strategy equilibrium, including Butters (1978), Varian (1980), and Stahl (1989).

4.3 Firm A's information is a proper subset of firm B's

There are two scenarios in this case: (a) Firm A learns both β's and firm B learns one β or does not learn, and (b) Firm A learns one β and firm B does not learn. In the first scenario, firm A has complete information, and firm B knows that firm A would not advertise if the buyer is not interested in the product. Due to the informational advantage, firm A's profit is strictly positive, while firm B's profit is zero.

Proposition 4. *Suppose $J_A = \{\beta_1, \beta_2\}$ and J_B is a strict susbet of J_A. In this case, q_B is observed by firm A. Given the realized q_A, q_B,*

(i) *If $q_B < c$, firm B does not advertise. If $q_A = 0$, then firm A does not advertise. If $q_A = 1$, firm A advertise (with probability 1) with price $p_A = 1$.*

(ii) *If $q_B \geq c$, firm B advertise with probability $1 - c/q_B$ and a pricing strategy H_B defined below. If $q_A = 0$, then firm A does not advertise. If $q_A = 1$, firm A advertise with probability 1, with the price distribution H_A defined below.*

$$H_A(p; q_B) = \begin{cases} 0, & \text{if } p < c/q_B, \\ 1 - \dfrac{c}{p \cdot q_B}, & \text{if } c/q_B \leq p < 1, \\ 1, & \text{if } p \geq 1. \end{cases}$$

$$H_B(p; q_B) = \begin{cases} 0, & \text{if } p < c/q_B, \\ \dfrac{1 - \dfrac{c}{p \cdot q_B}}{1 - \dfrac{c}{q_B}}, & \text{if } c/q_B \leq p < 1, \\ 1, & \text{if } p \geq 1. \end{cases}$$

(iii) *The expected profit for firm A is*

$$\pi_A = \int_0^c q_B(1-c)\, \mathrm{d}F_{q_B}(q_B) + \int_c^1 q_B(c/q_B - c)\, \mathrm{d}F_{q_B}(q_B).$$

The belief distribution F_{q_B} is determined by the information choice J_B. The expected profit for firm B is zero.

To provide some intuition about this result, first notice that the firm B's profit must be zero. Fix $q_A = 1$ and $q_B > c$. Suppose firm B earns a positive profit. Then, it is the case that firm B advertises with probability 1, as otherwise, the profit is zero. Since q_B is known to firm A, the lowest price firm B could charge is c/q_B. In this case, c/q_B covers the advertisement cost c, so firm A could earn a positive profit by charging p_A slightly less than c/q_B. It follows that firm A must advertise with probability 1 as well. Now consider the highest common price $\hat{p}$ both firms could charge. Both firms must place a mass on $\hat{p}$; if not, the winning probability vanishes as the prices converge from below, and thus, it is not profitable to advertise $\hat{p}$. When both firms tie at $\hat{p}$, it is strictly profitable for one to deviate to a low price, implying $\hat{p}$ is not the highest price the firm could charge.

Since the firm B must earn zero profit, the lowest price firm B charges is c/q_B. It is not profitable for firm A to charge any price below c/q_B, as firm A could win the competition of sales for sure at this price. The profit firm A earns is then $c/q_B - c > 0$. As firm A is indifferent between the prices in the support, pricing at 1 also earns the same profit. At $p_A = 1$, firm A wins only if firm B does not advertise. That is, $(1 - \gamma_B) \cdot 1 - c = c/q_B - c$, so the probability firm B advertises is $\gamma_B = 1 - c/q_B$.

Now consider the mass firm A places on the maximum price $p_A = 1$. Since firm B is indifferent among all the prices in the support, for p_B sufficiently close to 1, firm B wins with probability $1 - H_A(1)$. Because firm B earns zero profits, $(1 - H_A(1))q_B = c$, implying that the mass on top is c/q_B. As the firm B's belief q_B approaches the advertising cost, firm B is less likely to advertise, so firm A is more likely to charge the monopoly price.

Scenario (b) is more complicated, as firm B now knows that firm A would make mistakes. This could sometimes give room for both firms to earn positive profits. Suppose firm A learns β_1 and firm B does not learn. Like the previous scenario, firm A knows the prior belief is q_B. However, the informational advantage might not exist in this scenario.

Proposition 5. *Suppose firm A learns β_1 and firm B does not learn. Firm B's belief $q_B = 1/2$. If $\max\{q_A, 1/2\} < c$, then neither firm would advertise. If $1/2 < c \leq q_A$, then firm A advertises with probability 1 with $p_A = 1$, and firm B would not advertise. For the case, $\min\{q_A, 1/2\} \geq c$, consider the following condition*

$$\int_{q_A < \frac{1}{2}} q_A dF_{q_A}(q_A) = \mathbb{P}(q_A < 1/2)\mathbb{E}[q_A | q_A < 1/2] \leq c \tag{1}$$

(i) *If (1) holds, then firm A advertises with probability 1 if $q_A \geq q_B$ and does not advertise* *otherwise. The price offer when advertising follows the distribution*

$$H_A(p) = \begin{cases} 0, & \text{if } p < 2c, \\ \dfrac{\dfrac{1}{2} - \dfrac{c}{p}}{\displaystyle\int_{q_A > \frac{1}{2}} q_A \,\mathrm{d}F_{q_A}(q_A)}, & \text{if } 2c \leq p < 1 \\ 1, & \text{if } p \geq 1. \end{cases}$$

Firm B enters with probability $1 - 2c$*, and send a price following the distribution*

$$H_B(p) = \begin{cases} 0, & \text{if } p < 2c, \\ \dfrac{1 - \dfrac{2c}{p}}{1 - 2c}, & \text{if } c/q_B \leq p \leq 1 \\ 1, & \text{if } p \geq 1. \end{cases}$$

(ii) *Suppose* (1) *does not hold. Then firm A advertise4s whenever* $q_A \geq \tilde{q}$ *for some* $\tilde{q} > c$ *defined below. Firm A sends a price with distribution*

$$H_A(p) = \begin{cases} 0, & , \text{ if } p < \underline{p} \\ \dfrac{q_B - \dfrac{2c}{p \cdot \tilde{q}}}{\displaystyle\int_{q_A > \tilde{q}} q_A \,\mathrm{d}F_{q_A}(q_A)}, & \text{if } \underline{p} \leq p < 1 \\ 1, & \text{if } p \geq 1. \end{cases}$$

where $\underline{p}$ *and* $\tilde{q}$ *are defined by*

$$\begin{cases} \dfrac{1}{2} \cdot \underline{p} = \displaystyle\int_{q_A < \tilde{q}} q_A \,\mathrm{d}F_{q_A}(q_A), \\ \tilde{q} = \dfrac{c}{\underline{p}}. \end{cases}$$

Firm B's always advertises and send a price following the distribution

$$H_B(p) = \begin{cases} 0, & \text{if } p < \underline{p}, \\ 1 - \underline{p}/p, & \text{if } \underline{p} \leq p < 1 \\ 1, & \text{if } p \geq 1. \end{cases}$$

(iii) *The distribution of firm A's belief is uniform (0,1), i.e.,* $F_{q_A} = q_A$. *Therefore,* $\underline{p} = c^{2/3}$, *and* $\tilde{q} = c^{1/3}$. *The expected profit for the firm A is*

$$\pi_A = \begin{cases} \displaystyle\int_{c^{\frac{1}{3}}}^{1} \left(q_A\ c^{\frac{2}{3}} - c\right) \mathrm{d}q_A, & \textit{for } 0 \le c < 1/8, \\[2ex] \displaystyle\int_{1/2}^{1} (2q_A - 1)\, c\, \mathrm{d}q_A, & \textit{for } 1/8 \le c \le 1/2, \\[2ex] \displaystyle\int_{c}^{1} (q_A - c)\ \ \mathrm{d}q_A, & \textit{for } 1/2 < c \le 1. \end{cases}$$

The expected profit for firm B is

$$\pi_B = \begin{cases} \dfrac{c^{2/3}}{2} - c, & \textit{for } 0 \le c \le 1/8, \\ 0, & \textit{otherwise.} \end{cases}$$

(1) quantifies how information from firm A helps when comparing to cost of making mistakes. If the cost of making mistakes is low, advertising with probability 1 at the highest price could benefit firm B, as it is likely that firm A's belief can cover the cost, and thus, B will not use a mixed advertising strategy.

4.4 Characterization of the profits in the second-stage game

In Proposition 2, Proposition 3, Proposition 4, Proposition 5, I have stated the equilibrium under different information structures for the market segment $(1, 1)$. To complete the characterization of the profits in the second stage, I first introduce some notations. Let x be a market segment. Let $\pi_i^{\text{dist}}(x)$ be the expected profit for firm i in the market segment x where each firm learned non-overlapped information. Let $\pi_i^{\text{common}}(x)$ denote the firm i's expected profit whenever $J_A = J_B$. As argued below, similar to the analysis in Section 4.2, the expected profit is always zero, so this notation is well-defined. Let $\pi_i^{\text{full, partial}}(x)$ denote the expected profit for firm i, when $J_A = \{\beta_1, \beta_2\}$ and $J_B = \{\beta_1\}$ or $J_B = \{\beta_2\}$. Since the prior distribution for β_1 and β_2 are identical, the profit is the same regardless firm B learns β_1 or β_2. Let $\pi_i^{\text{partial, null}}(x)$ denote the expected profit for firm i, when $J_A = \{\beta_1\}$ or $J_A = \{\beta_2\}$, and $J_B = \emptyset$. The notations for the expected profits under other orders of the information choices, i.e., $J_B = \emptyset$, $J_A = \{\beta_1\}$ etc., are similarly defined.

Now, we connect the profit for other market segments to the results already obtained. Consider the market segment $x_1 0$ and J_A Learning only β_1 in x_{10} is equivalent to learning both characteristics in the market segment x_{11}.[5]. Therefore,

$$\pi_A^{\text{dist}}(x_{10}) = \pi_A^{\text{full, null}}(x_{11})$$

By Proposition 3, Proposition 4, whenever $J_i = \emptyset$ and $J_{-i} = \Omega$, or $J_i = J_{-i}$, the expected profit is zero. Combined with the above observation, I characterized the profit in the second stage in the following result:

Lemma 2. *Let $\Pi_i(J_A, J_B)$ denote the second-state aggregated profit of firm i under information choices J_A, J_B.*

(i) *Suppose $J_A = \{\beta_1\}$ and $J_B = \{\beta_2\}$. Then*

$$\Pi_A(J_A, J_B) = \Pi_B(J_A, J_B) = \frac{1}{3}\left[\pi_A^{\textit{full, null}}(x_{11}) + \pi_A^{\textit{dist}}(x_{11})\right].$$

(ii) *Suppose $J_A = \{\beta_1, \beta_2\}$, $J_B = \emptyset$. Then, $\Pi_A(J_A, J_B) = \pi_A^{\textit{full, null}}(x_{11})$ and $\Pi_B(J_A, J_B) = 0$.*

(iii) *Suppose $J_A = \{\beta_1, \beta_2\}$, $J_B = \{\beta_1\}$. Then*

$$\Pi_A(J_A, J_B) = \frac{1}{3}\left[\pi_A^{\textit{full, partial}}(x_{11}) + \pi_A^{\textit{full, null}}(x_{11})\right],$$

and

$$\Pi_B(J_A, J_B) = \frac{1}{3}\pi_A^{\textit{partial, full}}(x_{11})$$

(iv) *Suppose $J_A = \{\beta_1\}$ and $J_B = \emptyset$. Then*

$$\Pi_A(J_A, J_B) = \frac{1}{3}\left[\pi_A^{\textit{full, null}}(x_{11}) + \pi_A^{\textit{partial, null}}(x_{11})\right],$$

and

$$\Pi_B(J_A, J_B) = \frac{1}{3}\pi_A^{\textit{null, partial}}(x_{11}),$$

(v) *Whenever the information choices coincide, both firms earn zero profit.*

5 Strategic decision of information choices

Given the profit characterization in the last section, I now analyze the Nash equilibrium in the first stage of the game across different information costs and advertising costs.

Since each firm has four information choices, the first-stage game is a normal-form game that could be represented by a 4-by-4 matrix. In general, multiple equilibria could be either

[5] The equivalence relies on the assumption that β_1 and β_2 are symmetric around zero

pure strategy or mixed strategy. I first provide a characterization of pure-strategy Nash equilibrium.

Define $\Delta^{21}(J_B) = \Pi_A(\Omega, J_B) - \Pi_A(\{\beta_1\}, J_B)$. Δ measures the value of information fixing the opponent's strategy.

Proposition 6. *There exists a unique* $c^\star \in (\psi(1/2), 1)$ *such that* $\Pi_A(\{\beta_1\}, \emptyset, c^\star) = \Pi_A(\{\beta_1\}, \{\beta_2\}, c^\star)$. *For* $c > c^\star$,

(i) *If* $t = 0$, *all the NE are pure, and the set of equilibria includes all strategy profiles that at least one firm learns both characteristics.*

(ii) *For* $0 < t < \Delta^{21}(\{\beta_2\})$, *let* S_1 *by set of pure NE.* S_1 *consists of strategy profiles that one firm learns both covariates and the other does not learn:* $J_i = \Omega$ *and* $J_{-i} = \emptyset$

(iii) *For* $\Delta^{21}(\{\beta_2\}) < t < \Pi_A(\{1\}, \{2\}, c)$, *let* S_2 *be the set of pure NE consists of the strategy profiles. Then,* S_2 *consists of all the strategy profiles in* S_1 *and the strategy profiles that both firms learn distinct strategies, i.e.*

$$S_2 = S_1 \cup \{(\{\beta_1\}, \{\beta_2\}), (\{\beta_2\}, \{\beta_1\})\}.$$

(iv) *For* $\Pi_A(\{1\}, \{2\}, c) < t < \Pi_A(\{1, 2\}, \emptyset, c)/2$, *the set of pure NE is the same as* S_1.

(v) *For* $\Pi_A(\{1, 2\}, \emptyset, c) < 2t$, *there exists a unique NE where neither firm learns, i.e.,* $J_A = J_B = \emptyset$.

Note that the $\psi(1/2)$ denotes the cost corresponding to the cutoff of the bidding in the market segment (1,1), and both firm learns distinct features. $\psi(1/2)$ is around 0.2. For costs below $\psi(1/2)$, the profit could be high under other information structures due to mild consequences of incorrect advertising. Characterizing NE for those cases requires comparing the incentives to neglect punishment and charging a high price under different information structures; in other words, information is not valuable. In what follows, I focus on the $c > c^\star$ analysis, where $c^\star \approx 0.23$.

Part (i) says when there are no information costs, it is weakly dominant for one of the firms to choose full learning. Since it is costless to learn, any information choice is a best response to full learning.

Part (ii) says when the information cost is positive but less than $\Delta^{21}(\{\beta_2\})$, partial learning is not optimal. Consider one of the equilibria in (i), $J_A = \{\beta_1\}$ and $J_B = \Omega$. As claimed in Lemma 2, both common learning and partial learning lead to zero profit in the second stage. Hence, firm A prefers not to learn. Hence, the set of pure Nash equilibria features one firm learning fully and the other does not learn.

Note that although the profit of distinct learning is positive, it is not as high as of full learning, as the value of information is greater than the cost, i.e., $\Delta^{21}(\{\beta_2\}) > t$.

When the information cost exceeds $\Delta^{21}(\{\beta_2\})$, distinct learning generates more surplus than that of full learning. Hence, distinct learning creates new pure Nash equilibria. However, if one firm chooses full learning, the best response for its rival is still no learning.

Information cost in the range of (iv) is where partial learning is dominated by no learning. Full learning still generates a positive surplus, so the equilibria set is the same as in (ii). As information goes higher than the benefits of full learning, no learning is the only equilibrium.

There are multiple mixed-strategy equilibria in most cases. In what follows, I discuss how the equilibrium payoff of firm A changes, keeping one of the costs fixed and varying the other cost.

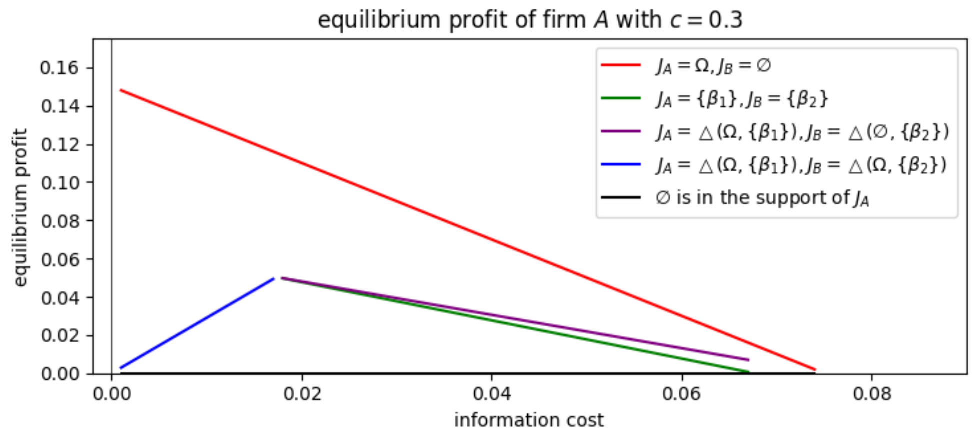

Figure I

THE EQUILIBRIUM PAYOFF OF FIRM A WHEN $c = 0.5$

Figure I plots the equilibrium profit of firm A when fixing the advertsing cost at $c = 0.3$. The red line is the equilibrium payoff of firm A when firm A chooses full learning and firm B chooses no learning. Since the information cost is sunk, the payoff is the fixed profit earned in the second stage minus the linear information cost. The blue line represents the mixed strategy equilibria that firm A is mixing between full-learning and learning β_1, and firm B is mixing between full-learning and learning β_2. These equilibria are symmetric in the sense the probability that each firm chooses distinct learning is identical. These equilibria only exist for $0 < t < \Delta^{21}(\{\beta_2\})$. In this case, whenever firm B learned β_2, it is optimal for firm A to learn both characteristics. To sustain these mixed strategy equilibria,

$$(1 - \alpha)\Delta^{21}(\{\beta_2\}) = t \tag{2}$$

where $1 - \alpha$ is the probability that firm A chooses β_1. The interpretation of (2) is not complex. For A to be indifferent between Ω and $\{\beta_1\}$, the value of information between full learning and distinct learning must be equal to the marginal information cost. The marginal information cost is always t. There is no value of information when firm B chooses Ω, hence only when firm B chooses to learn β_2 are there incentives for firm A to choose full learning. In equilibrium, the firm chooses to give up $(1-\alpha)$ of the value to cover the cost. As t goes up, firm A chooses to learn β_1 more frequently to sustain the equilibria. The payoffs of these equilibria are equal to the payoffs choosing to learn β_1, which is

$$(1-\alpha)\Pi_A(\{\beta_1\},\{\beta_2\}) - t = (1-\alpha)\left[\Pi_A(\{\beta_1\},\{\beta_2\}) - \Delta^{21}(\{\beta_2\})\right]$$

The first term in the bracket is gain from choosing to learn β_1. As long as it is higher than the value of information, firm A could gain from an increase in the information cost. This condition implies that under competition when information cost is low, the value of information given its rival learning partially is not that high. The main reason why this result holds is that firm A earns a large amount of profit in market segment $x = (1, 0)$.

As $t \geq \Delta^{21}(\{\beta_2\})$, there exist equilibria of distinct learning, indicated by the green line. The payoffs of these pure strategy equilibria decrease as t increases. There is also another set of mixed strategy equilibria where firm A mixes between Ω and β_1 and firm B mixes between $\emptyset$ and β_2, which is depicted by the purple line. The probability of choosing full learning increases, as information is more costly, firm B is more likely to choose no learning as its best response. The equilibrium payoff goes down as the payoff of choosing distinct learning becomes zero. The discontinuity happens when the payoff of distinct learning tends to zero. In this case, mixing between Ω and $\{\beta_1\}$ is dominated by no learning.

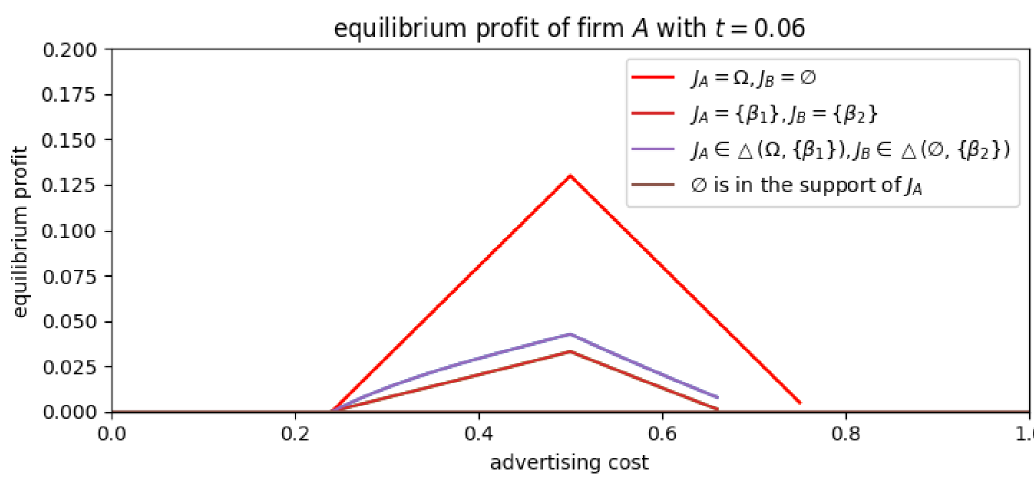

Figure II
THE EQUILIBRIUM PAYOFF OF FIRM A WHEN $t = 0.06$

Now we discuss another scenario that, shown in Figure II, in which the information cost is fixed at $t = 0.06$. This is where information is somewhat costly. A general trend of these curves is they are increasing in advertising costs and then decreasing. In other words, by fixing the information cost, the competition is relaxed when the advertising cost is low. This can be explained by the mixed strategy price competition. Note that $J_A = J_B$ is never an equilibria. Therefore, at least there is a positive probability that the learning strategy in equilibrium induces an outcome where firm A learns perfectly, and firm B learns nothing in one of the market segments. In this situation, our previous observation indicates the market outcome, in this case, is equivalent to the market outcome where firm A learns both characteristics, firm B learns none, and they compete in the market segment $x = (1, 1)$. This portion of the profit contributes to a large proportion of the aggregate market and thus how competition is relaxed when the advertising cost is low. By Proposition 4, firm B only advertises when the advertising cost is lower than its prior belief. In this case, due to lack of information, firm B randomizes between advertising or not. This means the expected payoff for firm B is zero. Thus, the lowest price firm B could advertise is $p_B = 2c$, to barely cover the advertising cost on average. Firm A, due to the advantage of full learning, always advertise. Moreover, firm A could offer the same lowest price. Since $p_A = 2c$ is certain to win the sale, the profit is $2c - c$. As the advertising cost increases, it makes the firm B less likely to advertise, and pricing with higher prices benefits the firm A. As the advertising cost becomes higher, firm A earns the monopoly profit, which shrinks to zero as the cost of advertising goes up.

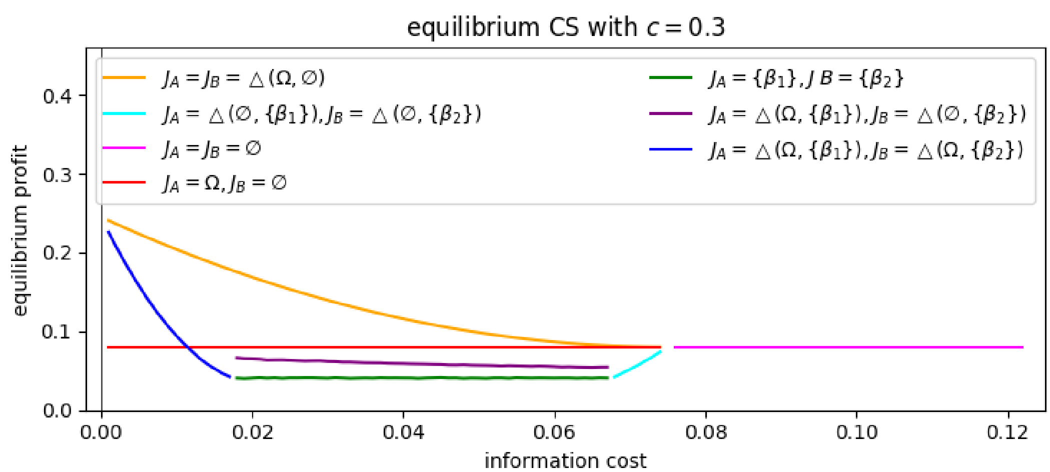

Figure III

THE EQUILIBRIUM CONSUMER SURPLUS WHEN $c = 0.3$

Figure III demonstrates the effect of information costs on consumer surplus. Since the information cost is sunk, for all pure Nash equilibria, the payoffs are represented by a horizontal line. For mixed strategy equilibria, consumer surplus generally decreases as information becomes more expensive. The key intuition here is consumer surplus is high whenever there is perfect price competition. When information cost is low, the equilibrium consumer surplus where both firms mix between full learning and no learning, represented by the yellow line in Figure III, is high. This is due to the fact that in this equilibrium, firms put high weight on full learning as it generates a high surplus when its rival does not choose high learning. But in equilibrium, there is a large probability both firms choose full learning, which yields a high consumer surplus. As information costs increase, competition is softened as firms are more likely to choose no learning. Similar intuition is observed in the other mixed equilibria where both firms mix between full learning and distinct learning.

As the information cost is relatively high, the only equilibrium is when both firms choose no learning, which yields a small, yet, positive consumer surplus.

6 Conclusion

Personalized pricing under competition depends on how firm would endogenously acquire consumer information. This paper studies discuss how endogenous acquisition of consumer data would affect market competition. As firms have acquired information, they are capable of offering personalized pricing. When there is a single seller, full learning is preferred when the information cost is neither too high nor too low, and no learning is optimal otherwise.

Under competition, in contrast, multiple equilibria arise. When both advertising cost and information are intermediate, there is an equilibrium outcome where both firms are coordinated to learn distinct consumer characteristics. If advertising cost is not too high (but high enough that firms are willing to learn to avoid loss), increasing in advertising cost relaxes the competition. The reason for less competition is that a large amount of profit is generated in a market segment where the equilibrium information choice leads to a situation in which one firm learns completely and the other firm is ignorant. Because it is costly to advertise, the ignorant firm randomly advertises some price. Due to ignorance, the lowest price the ignorant firm would charge is high, and there is an increase in the advertising cost. The informed firm can generate a positive profit by matching this price. Therefore, as the advertising cost increases, the ignorant firm advertises less frequently, and the market price rises, leading to a higher profit for the informed firm.

This work suggests several compelling directions for future research. The modeling choice in this paper encompasses the idea that firms are capable of perfectly acquiring information, and they could price discriminate buyers based on the observed characteristics. One direction is to build a data-based model to study personalized pricing under competition, as it would allow for effect due to the quality of the data. This paper also sheds some light on empirical models of personalized pricing, as the pricing observed might be an outcome of the strategic choice of information acquisition. A recent literature trend on endogenous information is to allow for flexible information acquisition. For theoretical direction, it would be interesting to relax the product differentiation and study flexible information acquisition under competition.

Appendices

A Omitted proofs

Proof of Proposition 2. I will prove the result by constructing the equilibrium directly. Let p_A and p_B be increasing strategies. First, consider the case $\tilde{q} < 1/2$. Suppose firm A's type is q_A. The probability of winning and $v = 1$ for firm A by reporting q is

$$\mathbb{P}(q \leq q_B) + \mathbb{P}(1 - q_A \leq q_B \leq \tilde{q}) = 1 - q + \tilde{q} - (1 - q_A)$$

The first term is because we assume the equilibrium strategy is increasing, and the second term is to take into account winning when firm B has low belief. Then, the profit of reporting q_A is

$$u_A(p_A(q)) = p_A(q)[1 - q + \tilde{q} - (1 - q_A)] - c \tag{3}$$

Taking derivative,

$$\frac{d\, u_A(p_A(q))}{dq} = p'_A(q)[1 - q + \tilde{q} - (1 - q_A)] - p_A(q)$$

In equilibrium, truthful reporting is required to be a best-response, the first-order condition implies:

$$p'_A(q)[1 - q_A + \tilde{q} - (1 - q_A)] - p_A(q_A) = 0 \iff \frac{p'_A(q_A)}{p_A(q_A)} = \frac{1}{\tilde{q}}. \tag{4}$$

It is straightforward to check the second-order condition is satisfied. The general solution to the differential equation (4) is obtained by integrating both sides:

$$\ln p_A(x) = \frac{Ax}{\tilde{q}} \iff p_A(x) = A \exp\left(\frac{x}{\tilde{q}}\right),$$

for some constant A. As long as $A > 0$, this solution is an increasing function. To pin down the constant A, since advertising is a cutoff strategy, the price at $q_A = 1$ must be equal to 1, as firm A with this belief only wins when firm B does not advertise. Hence,

$$p_A(1) = 1 \iff A \exp\left(\frac{1}{\tilde{q}}\right) = 1 \iff A = \exp\left(-\frac{1}{\tilde{q}}\right) > 0$$

Hence, the particular solution to is

$$p_A(q_A) = \exp\left(-\frac{1-x}{\tilde{q}}\right) \tag{5}$$

Now we pin down the cutoff $\tilde{q}$. Note that at $q_A = \tilde{q}$, the payoff is zero. If payoff at $\tilde{q}$ is above zero, there exists type $q'_A < \tilde{q}$ that would deviate and advertise with a price of $p_A(q'_A)$ and earn a strictly positive profit, which is better than not bidding. If payoff at $\tilde{q}$ is less than zero, firm A is better off by not advertising. Therefore,

$$u_A(p_A(\tilde{q})) = 0 \iff c = \tilde{q}\exp\left(1 - \frac{1}{\tilde{q}}\right) \tag{6}$$

Since RHS is a strictly increasing function of $\tilde{q}$, $\tilde{q}$ is fully determined by c.

Now we consider the case that $\tilde{q} < 1/2$. As argued in the main text the equilibria strategy p_A cannot be increasing on $[\tilde{q},\ 1/2]$, I impose an condition that p_A is symmetric around $1/2$ for $q_A \in [\tilde{q},\ 1-\tilde{q}]$. Now I construct a continuous strategy on $[\tilde{q}, 1/2]$, $[1/2, 1-\tilde{q}]$, and $[1-\tilde{q}, 1]$, repspectively. For $q_A \in [1-\tilde{q}, 1]$, the solution is the same as in the first part, as the effect of Winner's curse is the same as in the first part.

For $q_A \in [1/2, 1-\tilde{q}]$, firm A receives positive payoff only when $q_B \geq q_A$. Therefore, firm A's utility is

$$u_A(p_A(q)) = \mathbb{P}(q_B \geq q)p_A(q) - c = (1-q)p_A(q) - c$$

The first-order condition is

$$\frac{d\,u_A}{dq}(q) = (1-q)p'_A(q) = p_A(q)$$

The general solution to this differential equation is

$$p_A(x) = \frac{B}{1-x},$$

for some constant B. To ensure continuity, at $q_A = 1-\tilde{q}$, by (5),

$$\frac{B}{\tilde{q}} = \exp(-1) \iff B = \frac{\tilde{q}}{e}$$

Symmetry around $1/2$ implies that for $q_A \in [\tilde{q}, 1/2]$,

$$p_A(x) = \frac{B}{x}.$$

Similar to the argument in the first part, at $q_A = \tilde{q}$, the utility has to be zero. Therefore,

$$u_A(p_A(\tilde{q})) = 0 \iff c = B = \frac{\tilde{q}}{e}.$$

Again, as c is an increasing function $\tilde{q}$, $\tilde{q}$ is fully determined by c. To check there is no incentive for firm A to deviate when $q_A \in [\tilde{q},\, 1/2]$, note that firm A only wins when $q_B \geq 1-q_A$. Hence,

$$u_A(p_A(q)) = \mathbb{P}(q_B \geq 1-q)\frac{B}{q} - c = q\frac{B}{q} - c = B - c = 0.$$

There is no incentive for misreporting, as the utility is flat around the true type. Now the proof is complete. ∎

Proof of Proposition 3. Instead of verifying the equilibrium strategy claimed in the proposition, I will construct the equilibrium directly. It is straightforward to check that when $q < c$, both firms should never advertise. Suppose $q \geq c$. Firm A's utility of advertising with p_A given firm B's entry decision γ and price strategy H is as follows:

$$\pi_A(p_A, H; q) = (1-\alpha)(q \cdot p_A) + \alpha(1 - H(p_A)) \cdot q \cdot p_A - c$$

If firm A's entry decision is randomized, then the expected payoff is zero. Since the upper support is 1, the entry decision $\gamma = 1 - c/q$. From the indifference conditions for all prices in the support,

$$H(p) = \frac{1 - \dfrac{c}{pq}}{\gamma} = \frac{1 - \dfrac{c}{pq}}{1 - \dfrac{c}{q}}.$$

Hence, for all prices,

$$H(p) = \begin{cases} 0, & \text{if } p < c/q, \\ \dfrac{1 - \dfrac{c}{pq}}{1 - \dfrac{c}{vq}}, & \text{if } c/q_B \leq p < 1 \\ 1, & \text{if } p \geq 1. \end{cases}$$

∎

To prove Proposition 4, instead of directly verifying the equilibrium, I will construct the equilibrium by proving the following lemma.

Lemma 3. *Suppose firm A learns both β_1 and β_2. Firm B learns at most one attribute so that q_B is known to firm A. If $q_B < c$, then firm A will always send a price offer $p_A = 1$ whenever $q_A = 1$ and not send any offer if $q_A = 0$. Firm B will not send price offers. If $q_B \geq c$, there exists an equilibrium that the firm A always sends a price offer if $q_A = 1$, with a mixed pricing strategy with distribution H_A. Firm B will adopt a mixed strategy $\gamma_B \in (0, 1)$ of whether engaging in competition. Whenever the firm B makes a price offer, its pricing strategy is a mixed strategy following distribution H_B.*

Proof of Lemma 3. The first part is immediate. Suppose $q_B \geq c$. Assume that $q_A = 1$. Then firm A is payoff of sending price offer p_A given γ_B and H_B is

$$\begin{aligned} U_A(p_A; q_A = 1) &= (1-\alpha_B)(p_A - c) + (\alpha_B)(H_B(p_A))(-c) + (\alpha_B)((1 - H_B(p_A))(p_A - c) \\ &= (1 - \alpha_B \cdot H_B(p_A))p_A - c, \end{aligned} \tag{7}$$

and the payoff for firm B given H_A is

$$\begin{aligned} U_B(p_B; q_B) &= q_B[H_A(p_B)(-c) + (1 - H_A(p_B))(p_B - c)] + (1 - q_B)(-c) \\ &= q_B(1 - H_A(p_B))p_B - c \end{aligned} \tag{8}$$

By the indifference condition between sending and not sending, (8) must be equal to 0. Moreover, for all prices in the support of H_A, (8) remains the same. Both conditions together give:

$$\begin{gathered} q_B(1 - H_A(p))p = c \\ 1 - H_A(p) = \frac{c}{p \cdot q_B} \end{gathered}$$

Since $0 \leq H_A(p) \leq 1$, firm A's strategy is

$$H_A(p) = \begin{cases} 0, & \text{if } p < c/q_b, \\ 1 - \frac{c}{p \cdot q_B}, & \text{if } c/q_B \leq p < 1, \\ 1, & \text{if } p \geq 1. \end{cases}$$

That is, firm A's pricing strategy is supported $[c/q_B,\ v]$, with only one mass point on v. Similarly, by indifference condition, for all prices in the support of H_B, (7) remains the same. Note that firm H_B's support is also on $[c/q_B,\ v]$. In particular, the indifference conditions on the two boundaries give:

$$\gamma_B = 1 - \frac{c}{q_B}$$

Since $0 \leq \gamma_B \leq 1$, $q_B \geq c/v$ is necessary. Using the indifference condition for pricing and γ_B, (7) implies

$$(1 - \gamma_B \cdot H_B(p))p = \frac{c}{q_B}.$$

Thus, firm B's pricing strategy is

$$H_B(p) = \begin{cases} 0, & \text{if } p < c/q_B, \\ \dfrac{1 - \dfrac{c}{p \cdot q_B}}{1 - \dfrac{c}{1 \cdot q_B}}, & \text{if } p \geq c/q_B \\ 1, & \text{if } p \geq 1. \end{cases}$$

It is straightforward to verify that the support of H_B is $[c/q_B,\ 1]$ and there is no mass point. ■

Proof of Proposition 5. Let γ_B denote firm B's probability of entrance and H_B be its pricing strategy. Then firm A's utility of sending a price offer p given γ_B and H_B is

$$U_A(p_A, H_B; \gamma_B, q_A, q_B) = q_A[\gamma_B(1 - H_B(p_A))p_A + (1 - \gamma_B)p_A] - c.$$

Since firm B can never charge a price below c/q_B, firm A charges $p_A = c/q_B$ yields

$$q_A \cdot \frac{c}{q_B} - c$$

which is positive whenever $q_A \geq q_B$. Therefore, firm A will at least guarantee some profit by charging $p_A = c/q_B$ whenever $q_A > q_B \geq c$. If firm $\gamma_B \in (0, 1)$, then charging $p_A = 1$ is sometimes profitable, together with the indifference condition,

$$\gamma_B = 1 - \frac{c}{q_B},$$

which has the same value as the α_B in Proposition 5. Therefore, if firm A plays a mixed pricing strategy against $\gamma_B = 1 - c/q_B$ and H_B, then

$$H_B(p) = \begin{cases} 0, & \text{if } p < c/q_B, \\ \dfrac{1 - \dfrac{c}{p \cdot q_B}}{1 - \dfrac{c}{q_B}} & \text{if } c/q_B \leq p \leq 1 \\ 1 & \text{if } p \geq 1. \end{cases}$$

Suppose firm A does not send any price offer whenever $q_A < q_B$. Since q_A is private, it is not observed by firm B. Firm B needs to place a belief of q_A. Let $F_{q_A}(x)$ be the belief that $\mathbb{P}(q_A \leq x)$. Firm B will regard q_A as a posterior belief of $\mathbb{P}(v = 1|\beta_1)$. Therefore, the expected payoff of firm B given q_B and firm A's pricing strategy H_A is

$$\begin{aligned} U_B(p_B, H_A; q_B) &= \int_{q_A < q_B} [q_A p_B - c] \, \mathrm{d}F_{q_A}(q_A) + \int_{q_A > q_B} [q_A(1 - H_A(p_B))p_B - c] \, \mathrm{d}F_{q_A}(q_A) \\ &= p_B \mathbb{E}_{q_A}[q_A] - p_B \int_{q_A > q_B} [q_A H_A(p_B)] \, \mathrm{d}F_{q_A}(q_A) - c \end{aligned} \tag{9}$$

Since firm B will never price lower than c/q_B, whenever firm A makes a price offer, its price will never be below c/q_B as well. Hence, for $p_B = c/q_B$, firm B's utility is

$$U_B(c/q_B, H_A; q_B) = \frac{c}{q_B} \mathbb{E}\left[(q_A)\right] - c = \left(\frac{1}{q_B} \mathbb{E}\left[q_A(q_A) - 1\right] \right) c = 0$$

To see why the last equality is true, notice that $q_B = \mathbb{P}(v = 1)$ and $q_A = \mathbb{P}(v = 1|\beta_1)$. Hence, by the law of iterated expectation, $\mathbb{E}\left[q_A(q_A)\right] = q_B$. This implies that firm B is indifferent between entering the market or not. If firm A plays a mixed pricing strategy against H_A, the indifference condition implies $U_B(p_B, H_A; q_B) = 0$ for all prices p_B in the support. It follows from (9) that

$$\frac{c}{p_B} = \mathbb{E}\left[q_A[q_A]\right] - H_A(p_B) \int_{q_A > q_B} q_A \, \mathrm{d}F_{q_A}(q_A)$$

$$H_A(p_B) = \frac{q_B - \dfrac{c}{p_B}}{\displaystyle\int_{q_A > q_B} q_A \, \mathrm{d}F_{q_A}(q_A)}$$

for p_B in the support. Under the condition (1) $H_A(v) \leq 1$. Suppose the condition (1) does hold. Then the firm's equilibrium pricing strategy is

$$H_A(p) = \begin{cases} 0, & \text{if } p < c/q_B, \\ \dfrac{q_B - \dfrac{c}{p}}{\displaystyle\int_{q_A > q_B} q_A \, \mathrm{d}F_{q_A}(q_A)}, & \text{if } c/q_B \leq p < 1 \\ 1, & \text{if } p \geq 1. \end{cases}$$

The expected profit, in this case, is 0 for firm B, and strictly positive for firm A, as for large enough q_A, the profit is strictly positive by charging $p_A = c/q_B$.

If the condition (1) does not hold, then both firms will enter with probability 1. They both play a mixed strategy of pricing with common support $[\underline{p},\ v]$ and earn positive profits. For firm A, it will only enter the market if $q_A \geq \tilde{q}$ for some $\tilde{q}$ to be determined. In this case, the firm A's utility of sending a price offer given H_B is

$$U_A(p_A, H_B; q_A, q_B) = q_A(1 - H_B(p_A))p_A - c$$

The indifference conditions at $\underline{p}$ and v gives

$$\begin{aligned} q_A\underline{p} - c &= q_A(1 - H_B(p))p - c \\ H_B(p) &= 1 - \underline{p}/p \end{aligned}$$

Note that there is only one mass point at $\underline{p}$. Since U_A is increasing in type, at type $q_A = \tilde{q}$, firm A's utility is zero. Therefore,

$$\tilde{q} = \frac{c}{\underline{p}}.$$

The strategy H_B is

Similarly, for the firm B's utility of sending a price offer given H_A is

$$U_B(q_B, H_A; q_B, F_{q_A}) = p_B \mathbb{E}_{q_A}[q_A] - p_B \int_{q_A > \tilde{q_B}} [q_A H_A(p_B)] \, \mathrm{d}F_{q_A}(q_A) - c$$

If indifference conditions holds at $\underline{p}$ and v, then $\tilde{q}$ must satisfy

$$\begin{aligned} q_B - \int_{q_A \geq \tilde{q}} [q_A] \, \mathrm{d}F_{q_A}(q_A) - c &= \underline{p} \cdot q_B - c \\ q_B - \int_{q_A \geq \tilde{q}} [q_A] \, \mathrm{d}F_{q_A}(q_A) &= q_B \cdot \underline{p} \\ \int_{q_A < \tilde{q}} q_A \, \mathrm{d}F_{q_A}(q_A) &= q_B \cdot \underline{p} \\ \int_{q_A < \tilde{q}} q_A \, \mathrm{d}F_{q_A}(q_A) &= c \cdot \frac{q_B}{\tilde{q}}. \end{aligned} \tag{10}$$

Define $K_1(q) = \int_{q_A < q} q_A \, \mathrm{d}F_{q_A}(q_A)$ and $K_2(q) = c \cdot \frac{q_B}{q}$. Recall it is assumed that (1) does not hold in this case. $K_1(q_B)$ thus must be greater than c. Since K_1 is strictly increasing in q, if $\tilde{q} > q_B$, then LHS must be strictly greater than RHS and no such $\tilde{q}$ satisfies (10). Moreover, if $\tilde{q} > q_B$, $U_B < 0$, so B enters with probability one is not profitable. Moreover, $\tilde{q} \geq q^\star$, otherwise LHS of (10) is less than c/v and RHS is greater than c/v. Now consider K_1 and K_2 on $[q^\star,\ q/B]$. $K_1(q_B) = \gamma c/v$ for some $\gamma > 1$. K_2 is a continuous, decreasing function in q with $K_2(c/v) = q_B > c$. Therefore, $K_2(q^\star) < c$. Since $K_1(q^\star) > c > K_2(q^\star)$ and $K_1(q_B) > c = K_2(q_B)$, there exists a unique $\tilde{q} \in (q^\star, q_B)$ that satisfies (10).

To solve H_A, using the indifference conditions,

$$p_B q_B - p_B \int_{q_A > \tilde{q}} [q_A H_A(p_B)] \, \mathrm{d}F_{q_A}(q_A) - c = c \cdot \frac{q_B}{\tilde{q}} - c$$

To summarize, firm A enters whenever $q_A \geq \tilde{q}$ and its pricing strategy

$$H_A(p) = \begin{cases} 0, & \text{if } p < \underline{p}, \\ \dfrac{q_B - \dfrac{c}{p}\dfrac{q_B}{\tilde{q}}}{\displaystyle\int_{q_A > \tilde{q}} q_A \, \mathrm{d}F_{q_A}(q_A)}, & \text{if } \underline{p} \leq p < 1 \\ 1, & , \text{ if } p \geq 1. \end{cases}$$

Firm B enters if for $q_B > c/v$, and its pricing strategy is

$$H_B(p) = \begin{cases} 0, & \text{if } p < \underline{p}, \\ 1 - \underline{p}/p, & \text{if } \underline{p} \leq p < v \\ 1, & \text{if } p \geq v. \end{cases}$$

For all types $q_A > \tilde{q}$, firm A's profit is strictly positive. For firm B's profit, under different realizations of q_A, its profit could be positive or negative. Its expected profit is always positive. ■

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